AIAA 79-1104R

# Statistical Identification of Satellites' Heat Transfer Characteristics

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## Abstract

A PROBLEM sometimes referred to as the backward analysis of heat transfer in a satellite is treated. Given heat quantities input and/or generated, and the component temperatures, the heat transfer coefficients (HTC) for conduction and radiation are sought. The mathematical formulation results in an ill-conditioned, noisy set of linear equations. Solution methods for the system of equations are discussed.

## Contents

In order to insure that a satellite will function in the space environment, it is necessary to check that all component temperatures will remain in the admissible ranges. For this purpose, the approximate heat balance equations

$$Q_i = C_i \dot{T}_i + \Sigma_{i\neq j} C_{ij}(T_i - T_j) + \Sigma_{i\neq j} R_{ij}\sigma(T_i^4 - T_j^4) + R_{i0}\sigma(T_i^4 - T_0^4) \quad (1)$$

are solved[1] with respect to temperatures $T$, given $Q$ (heat input/generated) and HTC, viz. $C$ (heat capacity and conductive coupling) and $R$ (radiative coupling). Here $i$ and $j$ are satellite component numbers; $j=0$ stands for space; $Q_i$ [W] is the heat received and/or generated by $i$; $T_i$ [K] is the temperature of $i$; $\dot{T}_i$ [K/s] the time differential of $T_i$; $C_i$ [J/K] $i$'s heat capacity; $C_{ij}$ [W/K], $i\neq j$, $C_{ij}=C_{ji}$, the conduction parameter between $i$ and $j$; $R_{ij}$ [m$^2$], $i\neq j$, $R_{ij}=R_{ji}$ assumed, the radiation parameter between $i$ and $j$; and $\sigma$ [W/(m$^2$K$^4$)] the Stefan-Boltzmann constant. Note that $Q$, $T$, $C$, and $R$ are all nonnegative.

The HTC are determined in principle by the components' material and geometric configuration; in practice, however, the calculation is difficult to carry out due to the large number of factors involved. Therefore it has been proposed[2,3] that the HTC be evaluated from the experimental data that consist of heat generated $Q$ and component temperatures $T$. Such an approach is fairly popular in Japan.

Consider a construction as in Fig. 1, with the heat transfer network shown in Fig. 2. Suppose the construction is irradiated and is slowly changing its position against the ray source. If five measurements $k=0,\ldots,4$ of $Q$ and $T$ are made,

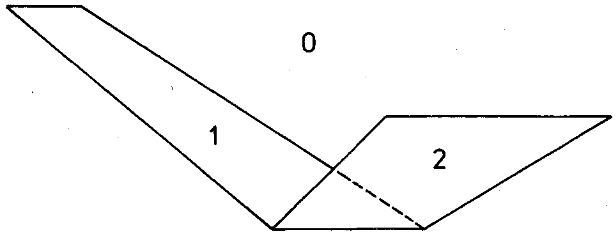


Fig. 1 A schematic construction.

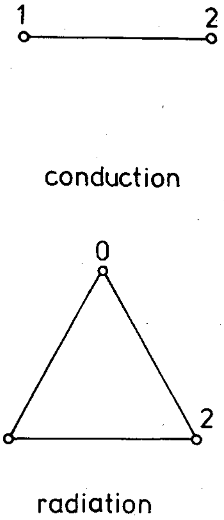


Fig. 2 Heat transfer structure for Fig. 1.

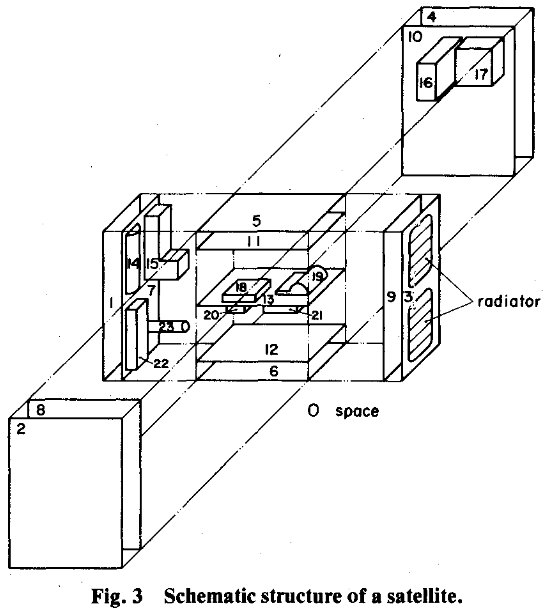


Fig. 3 Schematic structure of a satellite.

Presented as Paper 79-1104 at the AIAA 14th Thermophysics Conference, Orlando, Fla., June 4-6, 1979; submitted March 18, 1980; synoptic received July 17, 1980; revision received Oct. 3, 1980.  Full paper available from the AIAA Library, 555 W. 57th Street, New York, N.Y. 10019. Price: Microfiche, $3.00; hard copy, $7.00.

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the data structure may be expressed by

$$
\begin{array}{c}
k=1\\ \\ k=2\\ \\ k=3\\ \\ k=4\\ \\
\end{array}
\begin{bmatrix} Q_1^1 \\ Q_2^1 \\ Q_1^2 \\ Q_2^2 \\ Q_1^3 \\ Q_2^3 \\ Q_1^4 \\ Q_2^4 \end{bmatrix}
=
\left[\begin{array}{cc|c|ccc}
a_1^1 & & d_{12}^1 & q_{12}^1 & q_{10}^1 & \\
 & a_2^1 & -d_{12}^1 & -q_{12}^1 & & q_{20}^1 \\ \hline
a_1^2 & & d_{12}^2 & q_{12}^2 & q_{10}^2 & \\
 & a_2^2 & -d_{12}^2 & -q_{12}^2 & & q_{20}^2 \\ \hline
a_1^3 & & d_{12}^3 & q_{12}^3 & q_{10}^3 & \\
 & a_2^3 & -d_{12}^3 & -q_{12}^3 & & q_{20}^3 \\ \hline
a_1^4 & & d_{12}^4 & q_{12}^4 & q_{10}^4 & \\
 & a_2^4 & -d_{12}^4 & -q_{12}^4 & & q_{20}^4
\end{array}\right]
\left[\begin{array}{c} C_1 \\ C_2 \\ \hline C_{12} \\ \hline R_{12} \\ R_{10} \\ R_{20} \end{array}\right]
\begin{array}{l} \text{capacity} \\ \\ \text{conduction} \\ \\ \text{radiation} \\ \\ \end{array}
\qquad (2)
$$

where the superscripts represent measurement numbers. Here $a_i$ is a difference approximation of $\dot{T}_i$ such as

$$a_i^k = (T_i^k - T_i^{k-1})/(t^k - t^{k-1}) \qquad (3)$$

where $t^k$ is the time at which the $k$th measurement was made;

$$d_{ij}^k = U_i^k - U_j^k \qquad (4)$$

$$q_{ij}^k = \sigma[(U_i^k)^4 - (U_j^k)^4] \qquad (5)$$

where

$$U_i^k = (T_i^k - T_i^{k-1})/2 \qquad (6)$$

In real problems the number of components range from several to several hundreds; the number of unknowns from a score to several thousands. Consider a general system

$$y = Ax \qquad (7)$$

obtained as in Eq. (2). Typically, $A$ is very sparse. The system has no solution in the usual sense of linear algebra since it involves modeling and measurement errors. Thus it is convenient to solve Eq. (7) in the least-squares (LS) sense. The least-squares solution always exists but in most cases is not unique, because the rank of $A$ tends to be deficient when considered in contrast with the noise contained by its elements. This is perhaps due to the causality relationship that the explained variables $Q$ are actually causes; the explanatory variables $a$, $d$, and $q$ are functions of effects $T$.

The variable selection method (VSM), which sets some elements of $x$ equal to zero or to other given guesses, may be used to solve Eq. (7). If the LS problem is not to be solved directly but via the normal equation $A'y = A'Ax$, where the primes denote transposition, the conjugate gradient algorithm[4] may be adopted to make use of $A$'s sparsity. The usual statistical criteria for variable selection are not of much help since Eq. (7) does not fit into the standard regression model. Rather, the engineering guesswork turns out efficient.

When the problem is small, more sophisticated LS algorithms[5] such as singular value decomposition (SVD) and Marquardt's method may be employed. The experience with SVD proved, as expected, that it provides a far better LS solution than VSM, but a new difficulty arose: many elements of the solution $x$ were attributed negative values. In VSM this difficulty is solved implicitly since the engineer who operates the computer program to select variables knows that the solution must physically be nonnegative. In case a negative value appears, the engineer eventually deletes that variable. Algorithms are available to solve Eq. (7) under the nonnegativity condition,[5] but are time consuming.

VSM and SVD have been compared by simulation for a satellite model as shown in Fig. 3 which consists of 24 elements (with the space). The satellite is double walled like a thermal bottle; the heat generated in the inner container escapes to the space through the window in wall 3. Six steady state ($\dot{T}=0$) experiments in a space chamber have been assumed, with 136 nonzero HTC, 28 of which are for conduction and the rest for radiation. The accuracy of HTC estimates has been evaluated by the temperature residuals: observed temperatures minus predicted temperature using the HTC estimates. The VSM identified 23 $C$'s and 34 $R$'s; the temperature residuals were found to be of the order of 10 K. The SVD identified the rank of $A$ to be 105; the temperature residuals resulted on the order of 0.1 K.

Despite the difficulties we are optimistic with respect to the practicality of the method, because ancillary engineering information is always available in abundance. Systematic use of such a priori information should improve the estimation considerably.

## Acknowledgments

For the fundamental philosophy, the authors are indebted to Koichi Oshima of the Institute of Space and Aeronautical Science, University of Tokyo.

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